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2011 the 14th International Symposium on Wireless Personal Multimedia Communications (WPMC): 6081455-1-6081455-5

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Confenece Paper
A comparative study on MIMO MLSE turbo equalizer on frequency selective channels

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Abstract—In this paper, we propose an LDPC (Low Density Parity Check) coded MLSE turbo equalizer for the spatially multiplexed transmission on MIMO (Multiple Input Multiple Output) frequency selective channels. When the Channel State Information (CSI) is available at the receiver, Maximum Likelihood Sequence Estimation (MLSE) using Viterbi algorithm is effective to compensate the Inter Symbol Interference (ISI) and Inter Antenna Interference (IAI) on MIMO frequency selective channels [1]. We consider here the LDPC coded case where the reliable decoder output from LDPC decoder is fed back to the MLSE equalizer to enhance the ability of MLSE for compensating the ISI and IAI. The output from the MLSE equalizer is then LDPC decoded and its output is again fed back to the MLSE decoder. This feedback is repeated several times. We compared the proposing LDPC coded MIMO MLSE turbo equalizer with the conventional LDPC coded MIMO SC/MMSE (Soft Canceller with MMSE filter) turbo equalizer [2], [3] and compared their BER characteristics.

Keywords—MIMO; MLSE; LDPC; SC/MMSE; Turbo equalization

I. INTRODUCTION

Recently MIMO transmission scheme attracts much attention because of its high frequency efficiency and high reliability without expanding the transmission bandwidth. In SM (Spatially Multiplexed) transmission on MIMO frequency selective channels, the IAI, i.e., the interference caused by the multi transmit streams from multiple transmit antennas, and the ISI, i.e., the interference caused by delayed multipath channel between each transmit and receive antenna, have to be compensated at the receiver. For compensating the IAI and ISI at the receiver side, frequency domain or time domain method is considered. For frequency domain methods, the MIMO OFDM with CP (Cyclic Prefix) is well known and is widely used. For time domain methods, MIMO MLSE receiver using Viterbi algorithm was investigated and the BER characteristics are examined in [1]. It is well known that the MLSE equalizer has more equalization compensation ability than the FDE (Frequency Domain Equalization) because its nonlinear metric, while the FDE is the linear equalizer. In this paper, in order to further enhance the compensation ability of IAI and ISI, we have adopted the turbo equalization method to the MIMO MLSE with the error correcting code. We have used the LDPC code as the code with high error correction capability. Through the feedback of highly reliable decoded bits from the LDPC decoder to the MIMO MLSE equalizer, the compensation ability of IAI and ISI in MIMO MLSE equalizer is further improved. The soft outputs from the MLSE equalizer are then decoded by the LDPC decoder to give more reliable decoded outputs. These decoded outputs are again fed back to the MLSE equalizer, and the IAI and ISI are compensated repeatedly. Then, the outputs from MLSE equalizer are decoded by the LDPC decoder again. The above turbo feedback is repeated several times and finally the reliable LDPC decoded output is obtained. As the comparative scheme to the MIMO MLSE turbo equalizer, we have selected the conventional MIMO SC/MMSE turbo receiver [2], [3], and compared the BER characteristics between the two schemes. We have verified the effectiveness of LDPC coded MIMO MLSE turbo equalizer from the view point of BER characteristics.

II. LDPC CODED MIMO MLSE TURBO EQUALIZER

A. Transmit and Receive System Configuration

In Fig.1, we show the block diagram of transmit and receive system for the proposed LDPC coded MIMO MLSE turbo equalizer. At the transmitter, the LDPC coded information bits are SP (Serial to Parallel) converted and QAM modulated to be transmitted from each antenna. At the receiver, the received signals from each antenna are fed to the MIMO MLSE equalizer where the IAI and ISI are compensated to give the independent transmit streams. The bit LLR’s (Log Likelihood Ratio) from the MIMO MLSE equalizer are PS (Parallel to Serial) converted and fed to the LDPC decoder. The output bit LLR’s from the LDPC decoder are then fed back to MIMO MLSE equalizer and used to compensate the IAI and ISI again. The output of MIMO MLSE equalizer is again fed to the LDPC decoder. This feedback process is repeated several times to improve the BER iteratively.

B. MIMO MLSE equalizer

We denote the numbers of transmit antenna and receive antenna as \( n_T \) and \( n_R \) respectively. The channel model between each transmit and receive antenna is represented as the tapped delay line with the symbol delay time of \( T_s \). We also denote the number of delay path as \( L \). In the followings, we illustrate the case of BPSK modulation for the sake of brevity. When the transmit symbol at time \( t \) from transmit antenna \( i \) is
denoted as $x_i$ and the $l$-th delay path complex gain from the transmit antenna $i$ to the receive antenna $j$ is $h^{(l)(j)}i$ $(i=1,\ldots,n_i, j=1,\ldots,n_j)$, the receive signal $y_i$ at time $t$ from receive antenna $j$ is expressed as

$$y_i^j = \sum_{l=0}^{n_i} h^{(l)(j)}i x_i^j + n_i$$  (1)

In the trellis diagram of Fig.2, the numbers of states and branches at time $k$ are $2^{n_i(L-1)}$ and $2^{n_iL}$, respectively. Using the Viterbi algorithm to the trellis diagram, the MLSE can be done for the signal stream from each transmit antenna. However, the output from the Viterbi decoder have to be given by the bit LLR, i.e., soft value, thus bidirectional SOVA (Soft Output Viterbi Algorithm) (4) is employed instead of the usual Viterbi algorithm. In Fig.2, each branch connecting the state at time $t$ and the one at time $t+1$ has the branch metric given by

$$\nu_i(s_{s_i}, s_i) = \frac{1}{2\sigma^2} \left[ \sum_{l=0}^{n_i} y_i^j - \sum_{j=1}^{n_j} h^{(l)(j)}i x_i^j \right] - \sum_{l=0}^{n_i} \ln \Pr(x_i)$$  (2)

where the term $\ln \Pr(x_i)$ denotes the logarithm of priori probability and is given by the feedback from the LDPC decoder. The term is also expressed as

$$\ln \Pr(x_i^j = +1) = \Lambda(x_i) - \ln \frac{1}{1 + \Lambda(x_i)}$$  (3)

$$\ln \Pr(x_i^j = -1) = -\ln \frac{1}{1 + \Lambda(x_i)}$$

where $\Lambda(x_i)$ represents the bit LLR value fed back from the LDPC decoder output.

The path metric $\mu_i(s_i)$ in Fig.2 is evaluated as follows. As shown in Fig.2, the multiple branches emerging from the state at time $t = k-1$ are connected to the state at time $t = k$. The path metric $\mu_i(s_{s_i} \rightarrow s_i)$ at time $t = k$ is calculated as the sum of branch metric merging to the state and the path metric $\mu_i(s_{s_i})$ of the emerging state at time $t = k-1$. The survival path is defined as the path which has the minimum path metric and is selected among the paths merging to the state $s_i$ at time $t$, i.e.,

$$\mu_i(s_i) = \min \{ \mu_i(s_{s_i} \rightarrow s_i) \} = \min \{ \mu_i(s_{s_i}) + \nu_i(s_{s_i}, s_i) \}$$  (4)

The above path metric calculation is done to the forward direction ($t = 0 \rightarrow n$) and also to the backward direction ($t = n \rightarrow 0$). The path metrics calculated from the forward direction and the backward direction are denoted as $\mu^f_i$ and $\mu^b_i$ respectively. For each state $s_i$ at time $t$, the forward path metric $\mu^f_i$ and the backward path metric $\mu^b_i$ are added to produce the path metric $\mu_i$. The path metric $\mu_i$ at time $t$ which has the minimum value is denoted as $\mu_{min}^i$. Then we find the complementary path metric $\mu'_i$ competitive to the transmit signal which corresponds to $\mu_{min}^i$ at time $t$. The complementary path metric $\mu'_i$ is selected from the branches emerging from the state at time $t = 1$ and merging to $t$ with the different symbols assigned from $\mu_{min}^i$ and satisfies the minimization condition as shown below.

$$\mu'_i = \min \{ \mu^f_i(s_{s_i} \rightarrow s_i) + \nu'_i(s_{s_i}, s_i) + \mu^b_i(s_i) \}$$  (5)

When the receive sequence $Y'$ with the length of $r$, for which the bit LLR’s are evaluated is received, the posteriori probability of the transmit sequence $X'$ is given by

$$\Pr(X' | Y') = \Pr(X') \Pr(Y' | X') / \Pr(Y')$$  (6)

As the receive noise is white Gaussian, it holds

$$\Pr(Y' | X') = \prod_{i=1}^{n_1} \Pr(y_i | x_i, \ldots, x_{i(L-1)})$$

$$= \prod_{i=1}^{n_1} \frac{1}{\sqrt{2\pi}\sigma} \exp \{-\frac{1}{2\sigma^2} \left[ \sum_{j=1}^{n_j} y_i^j - \sum_{j=1}^{n_j} h^{(l)(j)}i x_i^j \right]^2 \}$$  (7)

By taking the logarithm at both sides of (7), we get

$$\ln \Pr(Y' | X') = \ln \prod_{i=1}^{n_1} \frac{1}{\sqrt{2\pi}\sigma} \exp \{-\frac{1}{2\sigma^2} \left[ \sum_{j=1}^{n_j} y_i^j - \sum_{j=1}^{n_j} h^{(l)(j)}i x_i^j \right]^2 \}$$

$$\equiv \sum_{i=1}^{n_1} \ln \frac{1}{\sqrt{2\pi}\sigma} \exp \{-\frac{1}{2\sigma^2} \left[ \sum_{j=1}^{n_j} y_i^j - \sum_{j=1}^{n_j} h^{(l)(j)}i x_i^j \right]^2 \} + r \ln \frac{1}{\sqrt{2\pi}\sigma}$$  (8)

We also get the logarithm of both sides of (6).

$$\ln \Pr(X' | Y') = \ln \{ \Pr(X') \Pr(Y' | X') / \Pr(Y') \}$$

$$= \sum_{i=1}^{n_1} \ln \frac{1}{\sqrt{2\pi}\sigma} \exp \{-\frac{1}{2\sigma^2} \left[ \sum_{j=1}^{n_j} y_i^j - \sum_{j=1}^{n_j} h^{(l)(j)}i x_i^j \right]^2 \} + \sum_{i=1}^{n_1} \ln \Pr(x_i)$$

$$+ r \ln \frac{1}{\sqrt{2\pi}\sigma} - \ln \Pr(Y')$$
Channel matrix:

\[
H' = \begin{bmatrix}
\hat{h}_0 & h_0' & \ldots & h_{L-1}' & 0 & \ldots & 0 \\
0 & h'_1 & \ldots & h'_{L-1} & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \ldots & \ldots & 0 & h'_L & \ldots & h'_{2L-1}
\end{bmatrix}
\]

Receive noise: \(N_i = [n_{i+L-1} \ldots n_i]^T, \quad n_i = [n_0^i \ldots n_n^i]\)

Using the above definition, the receive signal vector is expressed as

\[
Y_t = \sum_{i=1}^{L} H' x_i + N_i
\]

First, transmit signal at time \(t\) from transmit antenna 1 is detected. The soft replica signal used for the bit LLR’s fed back from the LDPC decoder.

\[
\hat{x}_c = E \{ x_i \} = \tanh \left( \frac{\lambda_i}{2} \right)
\]

Using the soft replica in (13), the IAI and ISI components are subtracted from the receive signal.

\[
\hat{y}_t = y_t - \sum_{i=1}^{L} H' \hat{x}_i
\]

The MMSE filter weight \(m_i'\) is determined so as to minimize the minimum mean square error given by

\[
m_i' = \min_{m_i} \text{MSE} = \min_{m_i} E \left\{ \left| x_i - m_i \hat{y}_t \right| \right\}
\]

\[
= \left( \sum_{i=1}^{L} H' A_i (H')^H + \sigma^2 I \right)^{-1} H'
\]

\[
A_i = \text{diag} \left[ 1 - \left| \hat{x}_{i+L-1} \right|^2, \ldots, 1 - \left| \hat{x}_{i-L+1} \right|^2, 1 - \left| \hat{x}_{i-L} \right|^2 \right]
\]

\[
|\hat{x}_{i-L}^2| = 0
\]

The output of MMSE filter \(z_i'\) is given by

\[
z_i' = (m_i')^H \hat{y}_t = \mu_i x_i + \eta
\]

where \(\mu_i^2\) and \(\eta\) denotes the amplitude and the noise of equivalent AWGN channel, respectively. \(\mu_i, E[\eta]\) and \(E[|\eta|^2]\) are expressed as

\[
\mu_i = (m_i')^H, \quad E[\eta] = 0, \quad E[|\eta|^2] = \mu_i (\mu_i')^2
\]

As mentioned above, the receive signal \(z_i'\) in the equivalent AWGN channel is obtained from the receive signal vector \(Y_t\) in the MIMO frequency selective channel. Using \(z_i'\), the output bit LLR \(\lambda_i'\) from the SC/MMSE equalizer is calculated as

\[
\lambda_i' = \ln \frac{\Pr \left( x_i' = +1 | z_i' \right)}{\Pr \left( x_i' = -1 | z_i' \right)} = \frac{4 \Re \left[ \mu_i z_i' \right]}{4 \Re \left( z_i' \right) - 1 - \mu_i'}
\]
IV. COMPUTER SIMULATION AND RESULTS

In Fig.5 ~ Fig.9, the simulation results of LDPC coded MIMO MLSE turbo equalizer and LDPC coded MIMO SC/MMSE turbo equalizer are shown. The computer simulation conditions for Fig.5, Fig.6 and Fig.7 are listed in Table I.

TABLE I. SIMULATION CONDITION FOR FIG.5, FIG.6 AND FIG.7

<table>
<thead>
<tr>
<th>Transmission schemes</th>
<th>LDPC Coded MIMO MLSE Turbo Equalization</th>
<th>LDPC Coded MIMO SC/MMSE Turbo Equalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulations</td>
<td>BPSK, or QPSK</td>
<td></td>
</tr>
<tr>
<td>Encoding</td>
<td>LDPC code</td>
<td>(1032,516)</td>
</tr>
<tr>
<td>Decoding</td>
<td>Log domain Sum Product</td>
<td></td>
</tr>
<tr>
<td>Sum-Product iteration</td>
<td>Up to 10 times</td>
<td></td>
</tr>
<tr>
<td>Antennas</td>
<td>Fig.5 and Fig.6: 2×2</td>
<td>Fig.7: 4×4</td>
</tr>
<tr>
<td>Fading between each transmit and receive antenna</td>
<td>Fig.5 and Fig.6: Quasi-static multipath fading having L=3 delay paths with equal average power</td>
<td>Fig.7: Quasi-static multipath Rayleigh fading having L=2 delay paths with equal average power</td>
</tr>
<tr>
<td>Equalizer</td>
<td>MLSE equalizer</td>
<td>SC/MMSE equalizer</td>
</tr>
<tr>
<td>Feedback Iteration</td>
<td>0,1,2,3</td>
<td></td>
</tr>
<tr>
<td>Channel state information</td>
<td>Perfect at receiver</td>
<td></td>
</tr>
</tbody>
</table>

Fig.5 and Fig.6 show the cases of BPSK and QPSK respectively for 2×2 antennas. The number of iterative feedback using the soft bit LLR value is set to 3. We examined the improvement and the convergence of BER. From Fig.5 and Fig.6, we can observe that the BER in quasi-static multipath fading is improved at each iterative feedback. Comparing with the MIMO SC/MMSE turbo equalizer, the BER of MIMO MLSE turbo equalizer at the 3rd iteration is improved by 1.0 dB and 1.6 dB at BER=10^{-4} for BPSK and QPSK respectively. Fig.7 shows the BER characteristics for quasi-static multipath fading with QPSK modulation for 4×4 antennas. The BER of MIMO MLSE turbo equalizer at the 3rd iteration is improved by 1.1 dB at BER=10^{-4} compared with the MIMO SC/MMSE turbo equalizer. The BER for MIMO SC/MMSE equalizer is improved largely for the 1st feedback. This is because no bit LLR is fed back to the SC/MMSE equalizer at the 0-th feedback, thus the SC/MMSE equalizer cannot generate the soft replica for the cancellation of IAI and ISI resulting in the poor BER, but at the 1st iteration the bit LLR is fed back to the SC/MMSE equalizer and the soft replicas can be generated leading to the large improvement of BER.

TABLE II. SIMULATION CONDITION FOR FIG.8

<table>
<thead>
<tr>
<th>Transmission schemes</th>
<th>LDPC Coded MIMO MLSE Turbo Equalization</th>
<th>LDPC Coded MIMO SC/MMSE Turbo Equalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulations</td>
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</tr>
<tr>
<td>Decoding</td>
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<tr>
<td>Sum-Product iteration</td>
<td>Up to 10 times</td>
<td></td>
</tr>
<tr>
<td>Antennas</td>
<td>2×2</td>
<td></td>
</tr>
<tr>
<td>Fading between each transmit and receive antenna</td>
<td>Quasi-static multipath fading having L=2 or 3 delay paths with equal average power</td>
<td></td>
</tr>
<tr>
<td>Equalizer</td>
<td>MLSE equalizer</td>
<td>SC/MMSE equalizer</td>
</tr>
<tr>
<td>Feedback Iteration</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Channel state information</td>
<td>Perfect at receiver</td>
<td></td>
</tr>
</tbody>
</table>

Next we examined the BER characteristics with the number of delayed multipath being changed. The simulation condition is given in Table II and the simulation results are given in Fig.8. As the BER almost converges at 3 iterative feedbacks, we use 3
feedbacks in the simulations hereafter. From Fig.8, in both MIMO MLSE turbo equalizer and MIMO SC/MMSE turbo equalizer, the BER is improved with the increasing number of delay paths. This is because of the diversity effect due to delay paths. The BER of MIMO MLSE turbo equalizer in case of \( L = 3 \) is improved by 1.6 dB at BER=10^{-3} compared with the MIMO SC/MMSE turbo equalizer.

Next we examined the BER characteristics when changing the numbers of transmit and receive antennas. The simulation condition is given in Table III and the simulation results are given in Fig.9. From Fig.9, in both MIMO MLSE turbo equalizer and MIMO SC/MMSE turbo equalizer, the BER is improved with the increasing number of transmit and receive antennas. This is because of the diversity effect due to the spatially paths increased. We also observe that the BER improvement of MLSE equalizer over the SC/MMSE equalizer in Fig.9 is large in the order of \( 2 \times 2 \), \( 4 \times 4 \) and \( 1 \times 1 \) antennas, but the BER difference in case of \( 1 \times 1 \) is very small, where the trellis length of MLSE corresponds to the LDPC code length.

V. CONCLUSIONS

In this paper, we proposed the LDPC coded MLSE turbo equalizer on MIMO frequency selective channels. In the proposed scheme, the reliable bit LLR from the LDPC decoder are fed back to the MIMO MLSE equalizer, where the bit LLR is used for the calculation of branch metric in the MIMO trellis diagram as the priori probability. We confirmed the BER improvement through the iterative feedback of LDPC decoded bit LLR. When comparing the BER characteristic with the conventional LDPC coded MIMO SC/MMSE turbo equalizer, the BER of MIMO MLSE turbo equalizer is superior to the MIMO SC/MMSE equalizer. In the proposed MIMO MLSE equalizer, the sequence estimation of transmit signal streams is done using the Viterbi algorithm in the MIMO trellis diagram, thus the number of trellis states increases exponentially with the numbers of transmit antennas and the delay paths. Accordingly reducing the complexity of MIMO MLSE equalizer will be the important future study.

Acknowledgment

This study has been supported by the Scientific Research Grant-in-aid of Japan No. 21560396, the A-STEP by JST (Japan Science and Technology Agency) and Sharp cooperation.

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