Global Noise Estimation Based on Tensor Product Expansion with Absolute Error

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SUMMARY This paper proposes a novel signal estimation method that uses a tensor product expansion. When a bivariable function, which is expressed by two-dimensional matrix, is subjected to conventional tensor product expansion, two single variable functions are calculated by minimizing the mean square error between the input vector and its outer product. A tensor product expansion is useful for feature extraction and signal compression, however, it is difficult to separate global noise from other signals. This paper shows that global noise, which is observed in almost all input signals, can be estimated by using a tensor product expansion where absolute error is used as the error function.

key words: global noise reduction, tensor product expansion, absolute error, electromagnetic wave, signal processing

1. Introduction

It is well known that multivariate analysis is useful for extracting the features of multivariate functions. The major method of extracting the characteristics of second-order statistics like principal component analysis (PCA) produces some global feature of various data. PCA is useful for feature extraction, however, it is difficult to separate global and local features raised by different sources. A tensor product expansion (TPE) [1], [2] can approximate an $m$-variable function as the sum of the product of $m$ single variable functions (SVFs). This technique is applied to nonlinear system identification [3], 3-D image processing [4], and achieve a substantial results, respectively. The tensor is calculated by minimizing a mean square error between the input vector and the sum of the product of SVF. Assume that an input signal expressed by 2-D matrix consists of observation global noises and local signals, the signal separation using TPE is difficult since local signals are treated as outliers. Main cause is that the separated signal is strongly affected by local signals due to mean squared error (MSE). If the new criterion, which can avoid the effect of outliers, is applied to TPE, the signal separation will be feasible. The absolute error is applied instead of MSE since outliers don’t influence the global noise estimation easily by using an absolute error. In this paper, we propose a method of a signal estimation that uses TPE and applies absolute error (TPE-AE) to separate a local signal from global noise; the former is observed in just few signals while the latter is seen in most signals.

This paper is organized as follows. In Sect. 2, tensor product expansion is explained. Section 3 gives a method of global noise estimation with a TPE-AE. The separability condition is also described. In Sect. 4, the relaxation method using a Monte Carlo simulation is proposed to solve a TPE-AE. In Sect. 5, some computer simulation results using an artificial signal and an electromagnetic wave are presented. Simulations show that proposed method is useful for estimating global noise.

2. Tensor Product Expansion (TPE)

Before we propose a method of signal estimation, tensor product expansion is introduced in this section. Assume that a bivariable function is expressed by 2-D matrix. Tensor and TPE are generalization of a matrix and singular value decomposition. As matrices are expanded into sums of vector products (tensor products), tensor can be expressed as sum of tensor products. This paper treats 2-D tensor since the signal observed with multi sensor system is represented by 2-D matrix. Let $F(l_1, l_2)$ be an element of 2-D tensor with indices of $l_1, l_2$. $F(l_1, l_2)$ can be expressed as:

$$F(l_1, l_2) = \sum_r f_r(l_1, l_2)$$

where $r$ is an index of iterative term, $f_1(l_1)$ and $f_2(l_2)$ are $l_1$th and $l_2$th element of a vector, respectively. (1) includes the product sum regarding $r$, however, this paper focuses on only the first term ($r=1$). Then, $f_1(l_1)$ and $f_2(l_2)$ are replaced by $f_{l_1}(l_1)$ and $f_{l_2}(l_2)$, respectively. $F(l_1, l_2)$ is obtained by the product of $f_{l_1}(l_1)$ and $f_{l_2}(l_2)$ that minimize a following evaluation function:

$$\sum_{l_1=0}^{q_1} \sum_{l_2=0}^{q_2} (h(l_1, l_2) - f_1(l_1) f_2(l_2))^2$$

where $h(l_1, l_2)$ is an input 2-D matrix consists of observation...
It is difficult that $f_1(l_1)f_2(l_2)$ in (3) approximates AC and DC components, since the equation does not have an additional term, which express a DC component. The following equation is proposed to obtain AC and DC components of $h(l_1,l_2)$:

$$
\sum_{l_1=0}^{q_1} \sum_{l_2=0}^{q_2} |h(l_1,l_2) - (f_1(l_1)f_2(l_2) + f_3(l_2))| \tag{4}
$$

where $f_1(l_1)f_2(l_2)$ and $f_3(l_2)$ are approximation terms of AC and DC components, respectively.

### 3.2 Separability Condition

The complete separability condition of the proposed method is not generalized yet due to a nonlinear criterion and complexity of the transformational proof. We discuss here one of the separability in the limited condition; the global noise is observed in almost signals while the local signal is caught in just few signals.

Assume that $sa(l_1)$ and $sb(l_1)$ are the source signal of global noise and a local signal, respectively, and $A_{12}$ and $B_{12}$ are a mixed matrix and DC component, respectively. A steady signal that includes only global noise is obtained by the following expression:

$$
A_{12}sa(l_1) + B_{12} \tag{5}
$$

and an unsteady signal that includes global noise and a local signal is given by:

$$
A_{12}sa(l_1) + sb(l_1) + B_{12}. \tag{6}
$$

In this section, we show that the optimal $f_1, f_2, f_3$, which minimize (4), is given by a linear expression of global noise. If the time course $f_1(l_1)$ is approximated as $sa(l_1)$ in the following expression, global noise can be separated from an observed signal.

$$
f_1(l_1) = sa(l_1). \tag{7}
$$

If the input matrix $h(l_1, l_2)$ consists of a steady and an unsteady signal, we have

$$
h(l_1,0) = A_0 sa(l_1) + B_0 \tag{8}
$$

$$
h(l_1,1) = A_1 sa(l_1) + sb(l_1) + B_1. \tag{9}
$$

These expressions mean the condition that both signal contains a global noise, one has a local signal. Substituting (8),(9) into (4), we get

$$
\sum_{l_1=0}^{q_1} |A_0 sa(l_1) + B_0 - (f_1(l_1)f_2(0) + f_3(0))| \\
+ \sum_{l_1=0}^{q_1} |A_1 sa(l_1) + sb(l_1) + B_1 - (f_1(l_1)f_2(1) + f_3(1))| \tag{10}
$$

where first (second) term is the absolute error of the steady (unsteady) signal. Here, we assume $A_1$ and $B_1$ can be estimated by $f_2(l_2)$, $f_3(l_2)$, respectively. (10) is expressed as:

$$
\sum_{l_1=0}^{q_1} |A_0 X(l_1) + |A_1 X(l_1) + sb(l_1)|. \tag{11}
$$

To obtain (11) we used the fact that $X(l_1) = sa(l_1) - f_1(l_1)$. (11) is expressed by sum of two absolute terms. Since optimal $X(l_1)$ is independent for arbitrary $l_1$, it is only necessary to estimate optimal $X(l_1)$ for all of $l_1$. Therefore, we have:

$$
|A_0 X(l_1)| + |A_1 X(l_1) + sb(l_1)|. \tag{12}
$$

(12) includes two absolute terms. Optimal $X(l_1)$, which minimize (12), is calculated in the following 3 cases.

**Case1.** $|A_0| = |A_1|$

Relationship between $X(l_1)$ and (12) is shown in Fig. 1(a) condition on $|A_0| = |A_1|$ and $A_1 \cdot sb(l_1) < 0$. Fig. 1(a) shows that minimum value of (12) is obtained as:

$$
0 \leq X(l_1) \leq -sb(l_1)/A_1. \tag{13}
$$

Then optimal $X(l_1)$ in (12) is given by the following expression:

$$
s_a(l_1) + sb(l_1)/A_1 \leq f_1(l_1) \leq s_a(l_1). \tag{14}
$$

Assume that $|A_0| = |A_1|$ and $A_1 \cdot sb(l_1) > 0$, we get:

$$
s_a(l_1) \leq f_1(l_1) \leq s_a(l_1) + sb(l_1)/A_1. \tag{15}
$$

Since (14) and (15) include a local signal, $f_1(l_1)$ is not estimated as $s_a(l_1)$ in this case.

**Case2.** $|A_0| < |A_1|$
represents $h$ represents $780$ means that $f$ \( x \) are included in the observed signal, we have

$$
\sum X(l_1) < \sum y \quad : \quad \left\{ \begin{array}{l}
\sum X(l_1) < \sum y \\
\sum X(l_1) > \sum y \\
\sum X(l_1) = \sum y
\end{array} \right. 
$$

(23) shows that TPE-AE can estimate global noise $s_d(l_1)$ that satisfies the following conditions.

- Mixed matrix $A_1$ and DC component $B_1$ can be estimated.
- Mixed matrix of steady and unsteady signals $A_x, A_y$ satisfies $\sum |A_x| > \sum |A_y|$.

If elements of mixed matrix are approximately equal, (23) means that the global signal can be estimated when the local signal is included with a few signals of $h(l_1, l_2)$. Thus, a local signal is separated from an unsteady signal since global noise is estimated as a tensor product.

4. Monte Carlo Simulation

It is difficult to obtain optimal vectors $f_1(l_1), f_2(l_2), f_3(l_2)$ using TPE-AE since the evaluation function (4) has a nonlinear term. In this paper, we use a relaxation method based on Monte Carlo simulation (MCS) to solve TPE-AE. MCS produces optimal solutions by extensive trials using random numbers. Optimal vectors, which minimize (4) are obtained by the following steps:

**STEP**

1. Initialize $f_1(l_1), f_2(l_2), f_3(l_2)$ using a small random number.

2. $f_{\text{new}}(l_1) = f_1(l_1) + \text{rand}(l_1)$.

3. In $l_1$, $\sum_i [h(l_1, l_2) - (f_1(l_1)f_2(l_2) + f_3(l_2))] > \sum_i [h(l_1, l_2) - (f_{\text{new}}(l_1)f_2(l_2) + f_3(l_2))]
   \quad$ is satisfied, so $f_1(l_1) = f_{\text{new}}(l_1)$.

4. Repeat step 2 and 3, 10 times.

5. $f_{\text{new}}(l_2) = f_2(l_2) + \text{rand}(l_2)$.

6. In $l_2$, $\sum_i [h(l_1, l_2) - (f_1(l_1)f_2(l_2) + f_3(l_2))] > \sum_i [h(l_1, l_2) - (f_1(l_1)f_{\text{new}}(l_2) + f_3(l_2))]
   \quad$ is satisfied, then $f_2(l_2) = f_{\text{new}}(l_2)$.

7. Repeat step 5 and 6, 10 times.

8. $f_{\text{new}}(l_2) = f_3(l_2) + \text{rand}(l_2)$.

9. In $l_2$, $\sum_i [h(l_1, l_2) - (f_1(l_1)f_2(l_2) + f_3(l_2))] > \sum_i [h(l_1, l_2) - (f_1(l_1)f_{\text{new}}(l_2) + f_{\text{new}}(l_2))]
   \quad$ is satisfied, then $f_3(l_2) = f_{\text{new}}(l_2)$.

10. Repeat step 8 and 9, 10 times.

where $\text{rand}()$ is white Gaussian random number based on normal distribution $N(0,1)$. The termination criteria should be decided for above iterative algorithm. According to empirical result, almost TPE is updated enough by 1000 update. Therefore, this algorithm is repeated at 1000 times.
5. Simulation

5.1 Known Functions

The simulation results of TPE-AE are shown to demonstrate its effectiveness for global noise estimation. First, a simple artificial signal based on known functions is subjected to TPE-AE. The artificial signal consists of a global noise component and a local signal, which are generated by a sine wave and a block pulse train, respectively. The separability condition on the input vector was confirmed in Sect. 3.2 when a global noise and a local signal are observed at almost signals and a few signals, respectively. Then we assume that few local signals are observed in the input signal. Table 1 and Table 2 list the conditions of the input matrix $h(l_1, l_2)$ shown in Fig. 2(b), where $l_1$ indicates the index of the time courses, $l_2$ is the index of the artificial signal A-E (0 to 4). The vertical axis shows the amplitude of an artificial signal, the horizontal axis indicates the cycle of sine wave ($l_1/576$). In the input matrix, 2 signals include a local signal in cycles 7 to 10.

Figure 2(a) shows the total absolute error between the input matrix and estimated $f_1(l_1)f_2(l_2)+f_3(l_2)$ for up to 1000 updates. In this simulation, $f_1(l_1)$, $f_2(l_2)$, $f_3(l_2)$ are updated enough since the error curve saturates at 1000 times. The tensor product of the 2D TPE[2] that minimize (2) is shown in Fig. 2(c), proposed TPE-AE is shown in Fig. 2(d) and the result of subtracting Fig. 2(d) from Fig. 2(b) is shown in Fig. 2(e).

Figure 2(c) shows the combination of the sine wave and block pulse. This means that separating global noise from a local signal is difficult with TPE. Fig. 2(d) shows that the sine wave is estimated by the proposed method, but Gaussian noise is also extracted.

From Fig. 2(e), a local signal can be detected by subtracting a tensor product from input matrix. Hence, it is shown that global noise and a local signal can be separated by using TPE-AE.

### Table 1 Conditions of the artificial signal.

<table>
<thead>
<tr>
<th>Global noise</th>
<th>Block pulse</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: $\sin(2\pi/576)/3.0+N(0.8,0.05^2)$</td>
<td>0.3</td>
</tr>
<tr>
<td>B: $\sin(2\pi/576)/3.0+N(1.0,0.05^2)$</td>
<td>0.6</td>
</tr>
<tr>
<td>C: $\sin(2\pi/576)/4.0+N(0.8,0.05^2)$</td>
<td></td>
</tr>
<tr>
<td>D: $\sin(2\pi/576)/5.0+N(1.0,0.05^2)$</td>
<td></td>
</tr>
<tr>
<td>E: $\sin(2\pi/576)/5.0+N(0.8,0.05^2)$</td>
<td></td>
</tr>
</tbody>
</table>

: $N(\mu, \sigma^2)$ means Gaussian noise $\mu$ is mean, $\sigma^2$ is variance.

### Table 2 Conditions of the input matrix and TPE-AE.

<table>
<thead>
<tr>
<th>Sampling number ($q_1$)</th>
<th>5760</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vector number ($q_2$)</td>
<td>5</td>
</tr>
<tr>
<td>The number of just sine waves</td>
<td>3</td>
</tr>
<tr>
<td>The number of vector including block pulse</td>
<td>2(B,D)</td>
</tr>
<tr>
<td>Update times</td>
<td>1000</td>
</tr>
</tbody>
</table>

5.2 Global Noise Reduction in ELF EM Wave

We show the effectiveness of TPE-AE in assessing actual observation signals. Attention is being placed on the electromagnetic (EM) waves that radiate from the earth’s crust in advance of earthquakes and volcanic actively [5]. Such EM waves observed in the extremely low frequency band include global noise created by lightning radiation from the...
tropics. This noise has a daily trend and similarity in all stations. Each observation station captured the east-west, north-south, and vertical components and averaged them over 6 second intervals. Thus 14400 data points were collected per day for each component. In this paper, we use the data averaged over 150 seconds for convenience (i.e. 576 points per day per component). If global noise in the EM waves can be estimated, a local signal from the earth’s crust and lightning in the near field can be detected adequately. The input matrix for applying TPE-AE is composed of 7 signals captured on July 5–6, 2003 by 7 antennas. Table 3 shows the conditions of the input matrix. The length of time course is 1152, the number of signals is 7 where 1 signal includes local radiation. Therefore, \( l_1 \) indicates the index of the time course, \( l_2 \) is the index of the observation signal 0 to 6. The two observed signals shown in Fig. 3(b) include a daily trend and local radiation at 0.7 and 1.6 days. The vertical axis shows the EM level (pT/\( \sqrt{\text{Hz}} \)), the horizontal axis indicates the day \( (l_1/576) \).

Figure 3(a) shows the total absolute error between the input matrix and estimated \( f_1(l_1)\) for up to 1000 updates. In this simulation, \( f_1(l_1), f_2(l_2), f_3(l_2) \) are updated enough since the error curve saturates at 1000 times. The result of the typical TPE that minimize (2) shown in Fig. 3(c) includes global noise and local radiation. On the other hand, the proposed method can estimate the daily global noise from Fig. 3(d). The local radiation observed at 0.7 and 1.6 days can be detected by subtracting Fig. 3(d) from Fig. 3(b) (Fig. 3(e)). These results show that TPE-AE is effective in estimating global noise in ELF EM waves.

### 6. Conclusion

In this paper, we proposed a signal estimation method based on tensor product expansion that uses absolute error. We clarified the separability condition for global noise estimation. Simulation results have shown the effectiveness of the proposed method for the global noise observed signals. Other simulation results have shown that the proposed method can reduce the daily trend observed in ELF EM waves. However, these simulations do not show the performance for the complex signal separation, since simple input matrices are applied to simulations. Remaining problems are to reduce Gaussian noise, to apply the proposal to other signals and to show the complete separability condition.

### Acknowledgment

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### References


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**Table 3 Conditions of the input matrix and TPE-AE.**

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling number ( (q_1) )</td>
<td>1152</td>
</tr>
<tr>
<td>Vector number ( (q_2) )</td>
<td>7</td>
</tr>
<tr>
<td>The number of just global noise</td>
<td>6</td>
</tr>
<tr>
<td>The number of vector including local radiation</td>
<td>1</td>
</tr>
<tr>
<td>Update times</td>
<td>1000</td>
</tr>
</tbody>
</table>

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**Fig. 3** Global noise estimation from electromagnetic waves. (a) Absolute error between an input matrix and an estimated signal. (b) Electromagnetic waves. (c) Estimated global noise \( f_1(l_1) \) by TPE. (d) Estimated global noise \( f_1(l_1) \) by TPE-AE. (e) Estimated local signal \( h(l_1, l_2) - (f_1(l_1)f_2(l_2) + f_3(l_2)) \) by TPE-AE.


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